Normalization of student grades by class performance

Shoba Ittyipe and Peter Zizler

Abstract

We normalize student GPA grades by their relative performance in the classes they have taken.

1 Introduction.

Students at universities take many courses to graduate. At some universities, most often larger institutions, the student course grade performance is a function of their classmates. The course grades are awarded "on a curve" adjusted to reflect an expected mean and standard deviation of the performance in the given class. Therefore, in such a case, the student's grade very much depends on their classmate's performance. Smaller institutions, Mount Royal University, might shy away from such grading on a curve and award student grades independent of the performance of their peers.

Whatever the scenario for the grade assignment is, it is conceivable that students might be awarded harsher grades due to the high performance of their classmates. If the particular class had higher scores on average, and another class had lower scores on average, this process would normalize these scores based on how an individual student scored relative to the mean value of that class.

It is of interest to generate a marker that would detect such aspects of grading. Such a marker could be used by an institution to detect to what extent, if any, the student grade awards are affected by the student peer performance. Once such adjusted grades are produced, the student might assess to what extent their overall GPA was affected by their peers. If the adjusted GPA is higher than the actual one, the student's grade performance might have been assessed more harshly due to having high-performing classmates. On the other hand, if the adjusted GPA was lower than the actual one, then the student's performance might have been overestimated due to lower-performing classmates.

We propose a method to produce adjusted student grades that would reflect their class performance and as a consequence, we could generate adjusted GPA based on the class performance of the student peers. These should not, of course, replace the actual grades, but should be only informative.

Suppose a student has the following 8 grades

TERM	CRN	SUBJ	CRSE	GRADE	SCORE
201501	19249	COMP	1103	B	3.0
201501	18950	COMP	1615	C+	2.3
201501	15057	GNED	1101	B	3.0
201501	19514	GNED	1103	B	3.0
201504	45407	CHIN	2217	B-	2.7
201504	48039	MATH	1505	B+	3.3
201504	51200	MGMT	2130	C	2.0
201601	16332	COMP	1501	A+	4.0

from the 8 courses they have taken. We have chosen a sample of only 8 courses to illustrate the procedure, understanding that a student would have needed to have 40 grades from all the courses they have taken in order to graduate. The z scores of the class performance of each course grade is found to be

Grade	B	C+	B	B	B-	B+	C	A+/A
z score	0.5214	0.1000	0.2980	-0.0479	-0.7271	0.5072	-0.7528	0.5318

The grade of A+ and A has the same score. The course grades the student obtained and the associated z scores are unrelated. For example, the first grade of B+ (3.0) was obtained in COMP 1103. The course grade value of 3.0 has some corresponding z score in that particular class and that z score happens to be $z = 0.5214$. In particular, suppose the class COMP 1103 had 40 students there (including our student). The course grades (as values) of all 40 students happened to be

$$\{t_i\}_{i=1}^{40}$$

Then the z score value of 0.5 is obtained as

$$0.5 = \frac{3.0 - \bar{t}}{\sigma_t}$$

where

$$\bar{t} = \frac{1}{40} \sum_{i=1}^{40} t_i \ ; \ \sigma_t = \sqrt{\frac{\sum_{i=1}^{40} \left(t_i - \bar{t}\right)^2}{39}}$$

If it happens that all the students in the class (including our student) obtained identical grade scores, for example pass and fail grade and all passed, then we set the z score value for our student in that class to be $z = 0$.

We want to adjust all the 8 grades of our student, based on the z scores from all the 8 classes. The second grade of C+ had the associated z score from the class (COMP 1615) $z = 0.1000$. The student was above average and the grade seems harsh, we expect a bump up. On the other hand, the 5th grade is B- but the associated z score from the class (CHIN 2217) is $z = -0.7271$, not too big. We expect a mild drop as a result of the adjusted grade. We now present the method to generated these adjusted scores.

2 Method.

A student has the following GPA grades in their n classes they have taken

$$\{X_i\}_{i=1}^n$$

Each grade X_i was taken in a class and the corresponding z score from the class performance is given by z_i. Thus each GPA grade X_i now has the associated z score z_i. Define

$$X_{\max} = \max\{X_i\}_{i=1}^n \ ; \ X_{\min} = \min\{X_i\}_{i=1}^n$$

$$z_{\max} = \max\{z_i\}_{i=1}^n \ ; \ z_{\min} = \min\{z_i\}_{i=1}^n$$

$$\overline{z} = \frac{1}{n}\sum_{i=1}^n z_i \ ; \ \sigma_z = \sqrt{\frac{\sum (z_i - \overline{z})^2}{n-1}}$$

Each student has z scores $\{z_i\}_{i=1}^n$ of their class performance attached to the corresponding GPA grade $\{X_i\}_{i=1}^n$. It is desirable to generate adjusted student GPA scores $\{Y_i\}_{i=1}^n$ that would reflect the z scores obtained in the respective classes. In particular we would like to have

$$\frac{Y_i - \overline{Y}}{\sigma_Y} = \frac{z_i - \overline{z}}{\sigma_z}$$

where

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^n Y_i \ ; \ \sigma_Y = \sqrt{\frac{\sum_{i=1}^n \left(Y_i - \overline{Y}\right)^2}{n-1}}$$

while keeping the property

$$Y_{\max} = X_{\max} \ ; \ Y_{\min} = X_{\min}$$

This guarantees the range of grades is not changed. To that end we define normalized GPA grades by class performance as

$$Y_i = \frac{X_{\min} + X_{\max}}{2} + \frac{X_{\max} - X_{\min}}{z_{\max} - z_{\min}}\left(z_i - \frac{z_{\max} + z_{\min}}{2}\right)$$

Result 1.1 The z score of Y_i in the set $\{Y_i\}_{i=1}^n$ is equal to the z score of z_i in the set $\{z_i\}_{i=1}^n$. In particular, for each $i \in \{1, \ldots, n\}$, we have

$$\frac{Y_i - \overline{Y}}{\sigma_Y} = \frac{z_i - \overline{z}}{\sigma_z}$$

Proof. Let

$$Y_i = a + b\left(z_i - c\right)$$

where

$$a = \frac{X_{\min} + X_{\max}}{2} \; ; \; b = \frac{X_{\max} - X_{\min}}{z_{\max} - z_{\min}} \; ; \; c = \frac{z_{\max} + z_{\min}}{2}$$

We have

$$\overline{Y} = a + b\left(\overline{z} - c\right)$$

thus

$$Y_i - \overline{Y} = b\left(z_i - \overline{z}\right)$$

Now

$$\frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2 = \frac{1}{n}b^2\sum_{i=1}^{n}\left(z_i - \overline{z}\right)^2$$

thus

$$\sigma_Y = b\sigma_z$$

We have

$$\frac{Y_i - \overline{Y}}{\sigma_Y} = \frac{b\left(z_i - \overline{z}\right)}{b\sigma_{z_i}} = \frac{\left(z_i - \overline{z}\right)}{\sigma_{z_i}}$$

♣

Result 1.2 We have

$$\overline{Y} = \frac{X_{\min} + X_{\max}}{2} + \frac{X_{\max} - X_{\min}}{z_{\max} - z_{\min}}\left(\overline{z} - \frac{z_{\max} + z_{\min}}{2}\right)$$

Furthermore suppose for some $a, b \geq 0$ with $a + b = 1$ we have

$$\overline{z} = az_{\max} + bz_{\min}$$

then

$$\overline{Y} = aX_{\max} + bX_{\min}$$

Proof

$$\begin{aligned}
\overline{Y} &= \frac{X_{\min} + X_{\max}}{2} + \frac{X_{\max} - X_{\min}}{z_{\max} - z_{\min}}\left(\overline{z} - \frac{z_{\max} + z_{\min}}{2}\right) \\
&= \frac{X_{\min} + X_{\max}}{2} + \frac{X_{\max} - X_{\min}}{z_{\max} - z_{\min}}\left(az_{\max} + bz_{\min} - \frac{z_{\max} + z_{\min}}{2}\right) \\
&= aX_{\max} + bX_{\min}
\end{aligned}$$

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Note that we can not determine, based on the above, whether $\overline{Y} > \overline{X}$ or $\overline{Y} < \overline{X}$.

3 Example.

We continue with the data presented in the introduction. We illustrate the grade adjustment procedure on this example. We recall the lowest grade is C and the highest grade is A+. The new adjusted grades will keep this grade range the same. The vector X of the actual grades is given by

X	=	3.3	2.3	3.0	3.0	2.7	3.3	2.0	4.0

and the z score vector for the original grades obtained from respective class performance, independent of these grades is given by

z	=	0.5214	0.1000	0.2980	-0.0479	-0.7271	0.5072	-0.7528	0.5318

We apply the procedure above and obtain

X	=	3.3	2.3	3.0	3.0	2.7	3.3	2.0	4.0
Y	=	3.9838	3.3277	3.6359	3.0975	2.0402	3.9616	2.0000	4.0000

yielding

Original	B	C+	B	B	B-	B+	C	A+/A
Adjusted	A-	B+	B+	B	C	A-	C	A+/A

A statistic of interest is the mean GPA score from all the 8 classes for our student, both actual and adjusted.

actual	2.9500
adjusted	3.2558

Our student has a higher adjusted mean GPA score than the actual (non-adjusted) mean GPA score from all the 8 classes. As a result, the student is informed that their overall grade performance during their studies was possibly assessed more harshly due to the higher performance of their classmates during their studies.

We have collected blind data by stripping student identification information altogether. We have used the data from students who graduated from the BCIS program in 2022 and 2023. For each student, all 40 course grades applied for graduation are used. Subsequently, for each grade that the student received in each section, all student grades from that class are considered to calculate the z-score for that class. The z-score generates values that can be negative or positive. A z-score value of zero means the student had an average score in that class. An above zero or below zero indicates above average or below average grades, respectively. For example the z-score value of $z = 0.5$ would indicate that a student class performance was half a standard deviation above the mean. Similarly, the z-score value of $z = -1.00$ would indicate that a student class performance was one standard deviation below the mean.

Also, it is noteworthy to indicate that the adjusted grade is computed using the maximum and minimum of the grades for the entire set. The final normalized grade for each student in each class is converted back to the original scale, by using the same range. Hence, the original range is preserved, although individual grades may change within that range.